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ARISTOTLE AND BOETHIUS: TWO THESES AND THEIR POSSIBILITIES[1]

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Abstract

There is a kind of logical theses that can be a cognitive problem. They are theses that are not tautologies and people tend to accept as absolutely correct. This is the case of theses such as those of Aristotle and Boethius. This paper tries to give an explanation of the reasons why this happens. The explanation is based on the theory of mental models. However, it also resorts to modal logic and the account of the ideas presented by Lenzen. Thus, relating the general framework of the theory of mental models to basic aspects of modal logic and this last account, a possible solution of the problem is proposed.

Keywords: *Aristotle's thesis; Boethius' thesis; mental models; modal logic; possibility.*

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Aristóteles y Boecio: dos tesis y sus posibilidades

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Resumen

Existe un tipo de tesis lógica que puede ser un problema cognitivo. Se trata de tesis que no son tautologías y que las personas tienden a aceptar como absolutamente correctas. Es el caso de tesis como las de Aristóteles o Boecio. Este trabajo trata de ofrecer una explicación de las razones por las que esto sucede. La explicación se basa en la teoría de los modelos mentales. No obstante, también recurre a la lógica modal y al análisis de los planteamientos presentados por Lenzen. Así, relacionando el marco general de la teoría de los modelos mentales con aspectos básicos de la lógica modal y con dicho análisis, se propone una posible solución al problema.

Palabras clave: *tesis de Aristóteles; tesis de Boecio; modelos mentales; lógica modal; posibilidad.*

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ARISTOTLE AND BOETHIUS: TWO THESES AND THEIR POSSIBILITIES

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I. Introduction

Aristotle's thesis is problematic from the cognitive point of view. People often deem it as correct. Nevertheless, from the perspective of classical logic, it is not always true, since it is not a tautology (see, e.g., López-Astorga, 2016a; Pfeifer, 2012). In the literature, in addition, Aristotle's thesis is usually addressed along with another thesis sharing logical characteristics with it: Boethius' thesis (see, e.g., Lenzen, 2019). Hence, the fact that this kind of theses is habitually accepted by individuals is a challenge for any cognitive theory at present.

This paper will be focused on one of those cognitive theories: the theory of mental models (e.g., Byrne & Johnson-Laird, 2020). Thus, it will be intended to explain the problem of the mentioned theses from this theory. Accounts following the theory of mental models and offering reasons why people can tend to admit theses such as those have already been presented (e.g., López-Astorga, 2016a; 2016b). Nonetheless, those accounts generally acknowledge their limitations and indicate that there are still dark points the theory needs to clarify (e.g., they admit that the general framework of the theory also allows giving accounts showing that people should reject both theses). In this way, the proposal of the present paper is to move forward in this direction and give a better explanation based on the theory of mental models.

To do that, it will also take modal logic into account. Lenzen (2019) dealt with the two aforementioned theses from this last type of logic. The result of that study was to offer restrictions for the theses based on the propositional logic provided by Leibniz. Those restrictions are important because, as claimed by Lenzen (2019), they allow accepting both theses under the framework of several modal logics. And this is in turn relevant because, although its proponents do not normally agree, the theory of mental models can be easily related to modal logic (e.g., López-Astorga, 2018). In fact, it has been stated even that the theory can be consistent with the requirement Fitting and Mendelsohn (1998) proposed for every modal logic, that is, to be coherent with the relations established from the modal perspective of the Aristotelian square of opposition (López-Astorga, 2020).

Accordingly, the aim of this paper is to give an account of the way the theory of mental models can understand the cognitive difficulties associated to the theses raised by Aristotle and Boethius from two points: 1) the relations that can be found between the theory and modal logic, and 2) the restrictions indicated by Lenzen (2019) for those theses. Thereby, the process will be as follows:

First, what the theses are exactly and the reasons why they are a cognitive problem will be described. Second, given that the theses include conditional relations, the manner the theory of mental models considers those relations will be addressed. Then, some links that can be laid down between this last theory and modal logic will be commented on. Next, the restrictions Lenzen (2019) formulated for the two theses will be indicated. Finally, it will be shown how Lenzen's restrictions can help the theory of mental models explain the cognitive difficulties the theses cause.

II. Two related theses

Both theses are well-known and, with different symbols, can be found in many works. The one of Aristotle can be expressed, for example, in this way:

(1) $\neg(p \rightarrow \neg p)$

Where '¬' stands for negation and '→' represents logical implication. On the other hand, that of Boethius can be captured by this formula:

(2) $(p \rightarrow q) \rightarrow \neg(p \rightarrow \neg q)$

McCall (1975; 2012) links formulae such as (1) and (2) to the historical view of the conditional starting with Chrysippus of Soli. As it is also well known, that view claims the necessity of a connection between the two clauses of the conditional. This is important, since, as pointed out, (1) and (2) are not tautological formulae in classical logic. So, they can only be accepted under an approach other than the classical. That approach can be, following McCall, the one of the connexive logic (see also, e.g., Lenzen, 2019).

However, what is most relevant now is just the fact that (1) and (2) are not tautological in classical logic. That means that there are cases in which they can be false. Given the material interpretation of the conditional, which is that of classical logic and customarily attributed, against Chrysippus' criterion, to Philo of Megara (e.g., Bocheński, 1963; O'Toole & Jennings, 2004), a conditional can be true if one of these conditions happen: 1) the antecedent does not hold or 2) the consequent holds. Therefore, that p is false is enough, both in (1) and in (2), for the two theses to be false. If p is false, (3) is true.

(3) $p \rightarrow \neg p$

And if (3) is false, (1) is true. On the other hand, if p is false, both (4) and (5) are true.

(4) $p \rightarrow q$

(5) $p \rightarrow \neg q$

Nevertheless, if (5) is true (6) is not.

(6) $\neg(p \rightarrow \neg q)$

But then (4) is true and (6) is false, which makes (2) false.

The real situation is, on the contrary, as indicated, that people usually consider sentences with formal structures such as those in (1) and (2) to be always true (note that there may not be difference between correction and truth for naïve individuals). It is not hard to think about common sense examples in this way. It is difficult to accept that a sentence such as "if you are a person, then you are not a person" is correct or true. Likewise, it is not easy to admit that a sentence such as "if you go to the city, you will not stay with your sister" is compatible with "if you go to the city, you will stay with your sister".

Undoubtedly, one might think that all of this is not a real problem from the cognitive perspective. It can be stated that the only point the previous account makes is that individuals do not use classical logic when reasoning. Clearly, that is not a problem at all for several contemporary theories, which reject the material interpretation of the conditional (see, e.g., O'Brien, 2014).

Nonetheless, even if the idea that classical logic and human reasoning are related in no way is right, it keeps being necessary to explain the mental mechanism why people decide to accept (1) and (2). Perhaps, the theory of mental models can solve these difficulties.

III. The theory of mental models and the possibilities of the conditional

The theory of mental models assumes that any sentence linked to another one by means of a connective refers to possibilities (e.g., Khemlani, Hinterecker, & Johnson-Laird, 2017). Given that the conditional relation is essential in both (1) and (2), this aspect of the theory will be described only referring to that relation. Thus, in the case of a sentence such as (4), its possibilities or models would be those included in (7).

(7) Possible (p & q) & Possible (not-p & q) & Possible (not-p & not-q)

The manner (7) expresses the possibilities is the usual way the latest version of the theory of mental models tends to do it (e.g., Khemlani, Byrne, & Johnson-Laird, 2018). It represents the possibilities that should be normally attributed to the conditional. However, the second and third possibilities in it (that is, not-p & q and not-p & not-q) are actually presuppositions under the framework of the theory (further information on this point is to be found in, e.g., Johnson-Laird & Ragni, 2019). On the other hand, although the possibilities in (7) appear to denote the rows in the truth table of the conditional in which this last connective is true, the theory of the mental models does not assign to the conditional exactly the same characteristics as classical logic. This is because of, at least, two reasons. First, only the first possibility in (7) (that is, p & q) is easy to recover by people when processing a conditional sentence. The other two possibilities require more cognitive activity (see also, e.g., Johnson-Laird, 2012). Second, several factors, including the meaning of the words used in the clauses, can cause the possibilities to be different. A typical example given by the theory in this regard can be (8).

(8) "If oxygen is present then may be a fire" (Johnson-Laird & Byrne, 2002, p. 663).

It is evident that the possibilities of (8) are not those in (7), but the following (see also, e.g., Table 4 in Johnson-Laird & Byrne, 2002):

(9) Possible (p & q) & Possible (p & not-q) & Possible (not-p & not-q)

Where ‘p’ means the fact that oxygen is present and ‘q’ refers to the fact that there is a fire.

A difference is clear between (7) and (9). They have a different second possibility. In (9), it is precisely the case in which the conditional is false in classical logic (p & not-q). Nevertheless, in (7), it is the only case that cannot be linked to (8): the case of a fire without oxygen.

But maybe what is most important now is that, from this approach, in principle, it is hard to understand the cognitive phenomena related to (1) and (2). Starting with (1), it can be said that, obviously, the four combinations for its only variable (p) are (10), (11), (12), and (13).

(10) p & not-p

(11) p & not-not-p = p & p

(12) not-p & not-p

(13) not-p & not-not-p = not-p & p

Combinations (10) and (13) have to be removed because they present contradictions. This is because the theory of mental models does not admit inconsistencies inside the models or possibilities (e.g., Johnson-Laird, Khemlani, & Goodwin, 2015a). On the other hand, given that (1) does not have thematic content, meaning cannot cause phenomena such as the one occurring with (8) and (9). So, taking into account that (3) is a conditional, and following what has been pointed out for (4) and (7), (11) should also be eliminated. Therefore, the final result is just (12), which can be expressed by means of possibility (14).

(14) Possible (not-p)

However, (14) is actually a possibility for (3), which should lead to accept that (3) is possible and, accordingly, to reject (1) (an explanation akin to this one can be found, e.g., in López-Astorga, 2016a).

As far as (2) is concerned, the situation is not better. The formula into its first brackets is (4). As said, the possibilities of (4) are the ones in (7). Regarding the second brackets, from all that has been said, it can be claimed that the possibilities for (5) can be those indicated in (15).

(15) Possible (p & not-q) & Possible (not-p & not-q) & Possible (not-p & q)

Two possibilities (not-p & not-q and not-p & q) match in (7) and (15). Hence, it is not impossible that (4) and (5) can be true at the same time. In this way, it cannot be stated, as (2), that (4) implies the negation of (5) (for more difficulties and possible explanations of (2) resorting to the theory of mental models, see, e.g., López-Astorga, 2016b).

Nonetheless, perhaps all of this can change if the theory of mental models is seen as a modal logic system. That has already been posed. As mentioned, it has been argued even that the theory can be compatible with the requirement Fitting and Mendelsohn (1998) raised for any system trying to be a modal logic. That requirement referred to the need to follow the relations expressed by the modal square of opposition, and the idea that the theory of mental models fulfills that requirement has been proposed by López-Astorga (2020). The next section continues to describe links between this last theory and modal logic.

IV. Mental models and System K

One of the relations that have been provided between the theory of mental models and modal logic has to do with a modal system as simple as System K (e.g., Kripke, 1963; 1965). In particular, it has been affirmed that the theory of mental models is absolutely compatible with System K (e.g, López-Astorga, 2018). Basically, the idea is as follows:

According to the theory of mental models, the models are 'conjunctions of possibilities' such as (7), (9), (14), or (15) (e.g., Johnson-Laird, Khemlani, & Goodwin, 2015b). Thus, the basic proposal is to transform those conjunctions of possibilities into well-formed formulae of modal logic. To do that, it is enough to deem the conjunctions in the models as logical conjunctions, and to make each possibility be under the scope of the modal operator of possibility. Thereby, for instance the result for (7) is (16).

(16) $\Diamond(p \wedge q) \wedge \Diamond(\neg p \wedge q) \wedge \Diamond(\neg p \wedge \neg q)$

Where '$\Diamond$' is the modal operator of possibility and '$\wedge$' stands for logical conjunction (formulae similar to (16) and the other modal formulae in this section built from possibilities of the theory of mental models can be found, e.g., in López-Astorga, 2018).

As it is well known, (16) indicates that there is at least a possible world in which (17) is true, there is at least a possible world in which (18) is true, and there is at least a possible world in which (19) is true.

(17) $p \wedge q$

(18) $\neg p \wedge q$

(19) $\neg p \wedge \neg q$

Likewise, the formula for (9) would be evident too:

(20) $\Diamond(p \wedge q) \wedge \Diamond(p \wedge \neg q) \wedge \Diamond(\neg p \wedge \neg q)$

The possible world in which (18) is true disappears in (20). However, instead, (20) adds that there is at least a possible world in which (21) is true.

(21) $p \wedge \neg q$

But proposals such as this one (i.e., proposals such as that of López-Astorga, 2018) do not seem to solve the difficulties this paper is addressing. Although formulae such as (16) and (20) follow all of the requirements corresponding to System K, that does not suffice to remove the cognitive problems of (1) and (2). (14) can be transformed into this formula:

(22) $\Diamond\neg p$

And, in a possible world in which (23) is true, (3) is true as well (and (1) is false).

(23) $\neg p$

On the other hand, the modal formula for (15) is (24).

(24) $\Diamond(p \wedge \neg q) \wedge \Diamond(\neg p \wedge \neg q) \wedge \Diamond(\neg p \wedge q)$

And (24) points out that there are possible worlds in which (18), (19), and (21) are true. Therefore, there are worlds in which (4) and (5) can be true at once, which in turn means that (2) is false in those worlds.

V. A restriction for (1) and (2)

But, if developments such as the one of Lenzen (2019) are taken into account, perhaps it is possible to eliminate the difficulties. Lenzen (2019) presents a detailed study of the propositional logic provided by Leibniz. Nevertheless, what is relevant of that study for the present paper is that it allows establishing a restriction for (1) and (2). With regard to (1), the restriction is captured by (25).

(25) If $\Diamond p$, then $\neg(p \Rightarrow \neg p)$

With other symbols, (25) is (LEIB 1) in Lenzen (2019). In it, '$\Rightarrow$' denotes strict implication, that is, what Carnap (1947) names 'L-implication'. That is a kind of implication that is true in all of the possible worlds (or, following Carnap's terminology, in all of the state-descriptions).

Regarding (2), the restriction is expressed in (26).

(26) If ◊p, then [(p ⇒ q) ⇒ ¬(p ⇒ ¬q)]

With other symbols, (26) is (LEIB 2) in Lenzen (2019).

In this way, the restriction is clear. In the case of (1), (25) reveals that (3) cannot be true in all of the possible worlds if there is at least a possible world in which p is true. On the other hand, (26) claims that, if there is at least a possible world in which p is true, then the fact that (4) is true in all of the possible worlds implies, in all of the possible worlds, that (5) is not true in all of the possible worlds (because (6) would be true in at least the same possible world as p). Obviously, the implications with 'if' and 'then' both in (25) and (26) are expressed in natural language because they do not require to be strict implications.

Because, according to Lenzen (2019), (25) and (26) can be accepted in different systems of modal logic, they can be keys to solve the problems of (1) and (2) from the theory of mental models. This issue is explored below. However, perhaps it is important to clarify a previous point before. Lenzen (2019) indicates that, from the point of view of one of the reviewers of his paper, (26) would not be a theorem in System K. It seems to be a theorem in systems such as T. One might think that this is a problem, since, in papers such as López-Astorga (2018), the modal system that is linked to the theory of mental models is just K. As it is well known, System T includes an additional axiom. That axiom provides that, if a formula is necessary, that is, true in all the possible worlds, that very formula is the case. Nevertheless, as it can be checked below, the fact that the mentioned axiom was correct would have no influence on the main arguments of the present paper.

VI. The possibility of p and the theory of mental models

Indeed, if Aristotle's thesis is not (1), but (25), it is not a problem that people accept it. (25) is correct, since, as said, if p can be true, (3) cannot be always true. In the same way, if Boethius' thesis is not (2), but (26), it can also be expected that individuals admit it. (26) is true because, as also indicated, if p can be true and (4) is always that, (5) is false at least when p is true.

Thus, the only point that needs to be clarified is how the fact that people understand (1) as (25) and (2) as (26) can be accounted for from the theory of mental models. Maybe that is easy to do. The concept of possibility implies that, when people have to think about the truth or falsity of a particular sentence, they analyze all of the possibilities that can be imagined given the information available (for the manner the theory of mental models considers a priori true and false sentences with thematic content, see, e.g., Quelhas, Rasga, & Johnson-Laird, 2017; 2019). However, if, as explained above, the possibilities are deemed as possible worlds in modal logic, the task is transformed into the activity to check all of the possible worlds related to (or accessible from) the real world.

This is important because the literature and the works supporting the theory of mental models (e.g., most of those cited in this paper) show that it is very unusual that individuals ignore the case in which the antecedent of a conditional is true. They always tend to pay attention to that possible scenario. It seems that the first clause of the conditional is irrelevant only in two situations: 1) when the conditional sentence is ironic and does not literally express what is stated in it, and 2) when the antecedent is undoubtedly false. An example of the first situation is (27).

(27) "If it works then I'll eat my hat" (Johnson-Laird & Byrne, 2002, p. 663).

It is obvious that (27) is an ironic sentence that does not allow taking into account any possible world in which the first clause occurs (for a more detailed explanation of how people often process sentences such as this one, see, e.g., Johnson-Laird & Byrne, 2002). With regard to the second situation, to find an example is not hard either:

(28) If elephants play video games, then triangles have three sides.

Beyond the consequences that (28) can have from the point of view of the material interpretation of the conditional, this kind of sentence usually leads to ignore worlds in which the antecedent is true as well. Except for fictional stories, people generally consider the first clause in sentences such as (28) false (that is, at least, what seems to be derived from the general literature on the theory of mental models).

However, apart from exceptional cases such as (27) and (28), as mentioned, the proponents of the theory of mental models appear to suggest that individuals habitually deal with the possible circumstances in which the antecedent is true (this can be affirmed, at a minimum, from studies as early as the one of Johnson-Laird & Byrne, 2002). So, they usually suppose possible worlds in which the antecedent happens. Nevertheless, if this is this manner, and, as also indicated, people have the tendency to consider all of the possible worlds when reflecting on the truth or falsity of a sentence, it is clear that they can understand (1) as (25). If they reflect on possible worlds in which the antecedent occurs, they will tend to assume that p is possible in Aristotle's thesis. Besides, if the worlds they analyze are all of the possible ones, they will also tend to deem the implication in that very thesis as a L-implication (and this apart from the fact that the analysis of all of the possible worlds can already imply the analysis of the worlds in which p is possible).

Likewise, it is not difficult to point out the reasons why individuals can interpret (2) as (26). If the antecedent of the two conditionals in Boethius' thesis (again, p) is not ignored, then they raise possible worlds in which it is true, that is, they suppose that p is possible. Given that, in addition, people usually pay attention to all of the possible worlds (which, by itself, as in the previous case, can also lead to assume that p is possible), they will have the trend to understand that the implications in this thesis are strict implications too.

VII. Conclusions

Nonetheless, if all of this is that way, it can be said that the theory of mental models can explain why individuals generally accept both theses. As indicated, it does not appear to be suitable to simply interpret that the acceptance of the theses demonstrate that the human mind works in a way different from the principles of classical logic. If it is argued that a mental process has nothing to do with that logic, one might expect at least an alternative explanation in that regard. However, looking for that alternative account, it is possible to come to interesting findings. That is what has happened in this paper, which has revealed the modal nature human logical reasoning might have.

It is true that, to develop the arguments above, it has been necessary to do something that the proponents of the theory of mental models usually explicitly reject: to link the theory to modal logic (see, e.g., Khemlani, Hinterecker, & Johnson-Laird, 2017). Nevertheless, the results that can be achieved from a relation of that type are clear both in the present paper and in others in the literature (e.g., López-Astorga, 2018).

Actually, the theory of mental models does not question the modal character of human cognition (e.g., Johnson-Laird & Ragni, 2019). As shown, the main concept of the theory is the one of possibility. In fact, it continuously tries to argue that reasoning is basically analyses of possibilities. What its adherents often deny is that those analyses have direct connection with logic (e.g., Johnson-Laird, 2010). Nonetheless, without challenging any other aspect of the theory of mental models, several works have not accepted this last idea (e.g., López-Astorga, 2018; 2020). From the point of view of those papers, it is possible to assume the theory and deem reasoning to be logic at once. The only point that should not be forgotten is that the logic that can be attributed to the human mind has to be essentially modal.

For these reasons, it seems that, in the study of cognition, the different historical and contemporary developments obtained from dealing with modal systems have to be taken into account. This appears to be relevant because it can offer conceptual tools and resources to understand phenomena such as those reviewed here. Hence, the sense of the research that must continue to be done in this regard is evident. It is important to keep going over the relations that can be provided between frameworks such as the theory of mental models, the experimental results in the literature, and what different modal logical systems can suggest.

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